LIMITS

Informal Definition of a Limit: The behavior of $f(x)$ as x approaches a value c, from left and right.

$$\lim_{x\to c} f(x) = L$$

Formal Definition of a Limit: Let f be defined on an open interval containing c (except possibly at c), and let L be a real number. The statement

$$\lim_{x\to c} f(x) = L$$

means that for every $\varepsilon > 0$, there exists a $\delta > 0$,
such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

Properties of Limits:

$\lim_{x\to a}[c \cdot f(x)] = c \cdot \lim_{x\to a} f(x)$

$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

$\lim_{x\to a}[f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \lim_{x\to a} g(x) \neq 0$

$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$

Limit Evaluations at Infinity for Rational Functions:

$R(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials.

1. If degree of $p(x)$ < degree of $q(x)$, then $\lim_{x\to -\pm\infty} \frac{p(x)}{q(x)} = 0$.
2. If degree of $p(x)$ = degree of $q(x)$, then $\lim_{x\to -\pm\infty} \frac{p(x)}{q(x)} =$ ratio of the leading coefficient.
3. If degree of $p(x)$ > degree of $q(x)$, then $\lim_{x\to -\pm\infty} \frac{p(x)}{q(x)} = \infty \; or -\infty$, depending on the sign of the leading coefficient.

Continuity at a point: A function $f(x)$ is continuous at a point if $\lim_{x\to c} f(x)$ exists, $f(c)$ is define and $\lim_{x\to c} f(x) = f(c)$.

Derivative definition:

$\frac{d}{dx}\big(f(x)\big) = f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$ slope of the tangent line

DERIVATIVES

Basic Properties:

$\big(c \cdot f(x)\big)' = c \cdot f'(x)$

$\big(f(x) \pm g(x)\big)' = f'(x) \pm g'(x)$

$\frac{d}{dx}(constant) = 0$

Power Rule:

$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$

Product Rule:

$\big(f(x) \cdot g(x)\big)' = f'(x)g(x) + f(x)g'(x)$

Quotient Rule:

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule:

$\frac{d}{dx}\Big(f\big(g(x)\big)\Big) = f'(g(x)) \cdot g'(x)$

Common Derivatives:

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$

INTEGRALS

Definite Integral Definition:

$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$, where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$

Fundamental Theorem of Calculus:

$\int_a^b f(x)dx = F(a) - F(b)$ where f is continuous on $[a, b]$ and $F' = f$.

Integration Properties:

$\int_a^b cf(x)dx = c\int_a^b f(x)dx$

$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

$\int_a^a f(x)dx = 0$ and $\int_a^b f(x)dx = -\int_b^a f(x)dx$

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

Power rule for Integration:

$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$

Common Integrals:

$\int \cos x\,dx = \sin x + C$

$\int \sin x\,dx = -\cos x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \sec x\,dx = \sec x \tan x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int e^x dx = e^x + C$

$\int \frac{1}{x}dx = \ln|x| + C$

Integration by Substitution:

$\int_a^b f\big(g(x)\big)dx = \int_{g(a)}^{g(b)} f(u)du$, where $u = g(x)$ and $du = g'(x)dx$

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