Final exam review – Math 2402

1. Find the derivative of the function.

   1a. $f(x) = \ln\left(\frac{2x}{x+3}\right)$

   1b. $g(x) = x^2 \ln x$

   1c. $h(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

2. Use L'Hopital's rule to find the limit.

   2a. $\lim_{x\to\infty} \frac{2x^2+3x+1}{x^2-3}$

   2b. $\lim_{x\to\infty} \frac{x^2-9}{e^x}$

   2c. $\lim_{x\to 0} \frac{e^{4x}-1-4x}{x^2}$

   2d. $\lim_{x\to 0} \frac{x-\ln(x+1)}{x^2}$

3. Find the first derivative of the following functions.

   3a. $f(x) = x^3 e^x$

   3b. $y = e^x(\sin x + \cos x)$

   3c. $y = \frac{e^x+1}{e^x-1}$

   3d. $f(x) = 4^{\pi x^2}$

   3e. $f(x) = 4^3$

   3f. $f(x) = x^x$

4. Find the indefinite integral.

4a. $\int \frac{9}{5-4x}\,dx$

4b. $\int \frac{x^2}{5-x^3}\,dx$

4c. $\int \frac{(\ln x)^2}{x}\,dx$

4d. $\int e^{-x^4}\,x^3\,\,dx$

4e. $\int 4^x\,dx$

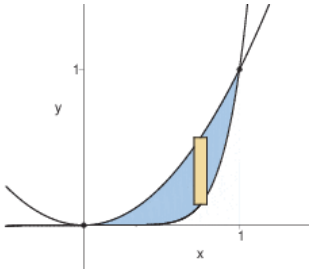
5. Find the derivative for the following trigonometric functions.

5a. $g(x) = 7^{\tan x}$

5b. $f(x) = x^2\,\arcsin(x)$

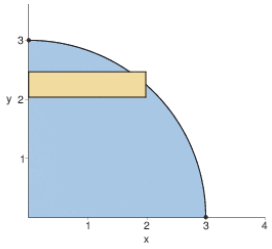
6. Write and evaluate the definite integral that represents the volume of the solid formed by revolving the region about the x-axis.

$$y = x^2, \quad y = x^7$$



7. Write and evaluate the definite integral that represents the volume of the solid formed by revolving the region about the y-axis.

$$y = \sqrt{9 - x^2}$$



8. Find the indefinite integral.

8a. $\int \frac{2}{e^{-x}+1}\, dx$

8b. $\int \frac{1}{25+4x^2}\, dx$

8c. $\int x^3 e^x dx$

8d. $\int x^5 \ln 3x \;\; dx$

8e. $\int e^{-3x} \sin 5x \;\; dx$

8f. $\int \arctan x \;\; dx$

9. Find an indefinite integral.

9a. $\int \cos^5 x \sin x \;\; dx$

9b. $\int \cos^4 x \;\; dx$

9c. $\int \sin^3 2\theta \sqrt{\cos 2\theta} \;\; d\theta$

9d. $\int \sec^4 2x \, dx$

9e. $\int \tan^3 \frac{\pi x}{2} \sec^2 \frac{\pi x}{2}\, dx$

9f. $\int \frac{4}{x^2\sqrt{16-x^2}} dx$

9g. $\int \frac{9x^3}{\sqrt{1+x^2}}\, dx$

9h. $\int e^x \sqrt{1-e^{2x}}\, dx$

9i. $\int \frac{x^2-6x+2}{x^3+2x^2+x} dx$

9j. $\int \frac{x^2}{x^4-2x^2-8} dx$

9k. $\int \frac{e^x}{(e^x-1)(e^x+4)} dx$

10. Determine whether the integral converges or diverges. If it is convergent find its value.

10a. $\int_e^\infty \frac{dx}{x\sqrt{\ln x}}$

10b. $\int_0^3 \frac{1}{\sqrt{3-x}} dx$

11. Confirm that the Integral Test can be applied to the series. Then use the Integral Test to determine the convergence or divergence of the series.

11a. $\sum_{n=1}^\infty \frac{1}{2^n}$

11b. $\sum_{n=1}^{\infty} \frac{4n}{2n^2+1}$

12. Determine whether each series is a p −series

12a. $\sum_{n=1}^{\infty} \frac{1}{n^{1.4}}$

12b. $\sum_{n=1}^{\infty} \frac{1}{n^{-2}}$

13. Determine the convergence or divergence of series.

13a. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

13b. $\sum_{n=1}^{\infty} \frac{3}{n^{\frac{5}{3}}}$

13c. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \frac{1}{5\sqrt{5}} + \cdots$

13d. $\sum_{n=1}^{\infty} \frac{1}{3n^2+2}$

13e. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$

13f. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

13g. $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$

14. Find the radius of convergence.

14a. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{5^n}$

14b. $\sum_{n=0}^{\infty} \frac{(4x)^n}{n^2}$

15. Find the Taylor polynomial P_4 for $f(x) = \ln(x)$ centered at $x = 1$.