

# Lightpath routing for maximum reliability in optical mesh networks

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We consider the problem of maximizing the reliability of connections in optical mesh networks against simultaneous failures of multiple fiber links that belong to a shared-risk link group (SRLG). We study the single-lightpath, parallel-lightpaths, and lightpath protection problems for connections between two end nodes, as well as the lightpath-ring problems for connections of three or more end nodes. We first study the special problems where all SRLGs have the same failure probability. In these problems, every SRLG is represented by a distinct color and every fiber link is associated with one or more colors, depending on the SRLGs to which the link belongs. We formulate the problems as minimum-color lightpath problems. By minimizing the number of colors on the lightpaths, the failure probability of the lightpaths can be minimized. We prove the problems to be NP-hard. We then extend the results to the general problems where the failure probabilities of the SRLGs may differ. Heuristic algorithms are proposed for larger instances of the problems, and the heuristics are evaluated through simulations. © 2008 Optical Society of America

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## 1. Introduction

In mesh WDM networks, users communicate with each other via end-to-end lightpaths [1,2]. Three types of connections are commonly used for communication between two end nodes: a single lightpath, multiple parallel lightpaths, and a working lightpath–protection lightpath pair. A single lightpath may reach a data rate of 40 Gbits/s or even higher [3,4]. Two or more parallel lightpaths are often used between backbone routers to carry traffic simultaneously, resulting in a higher aggregate data rate, with the additional benefits of load balancing [5]. The parallel lightpaths are link disjoint so that the failure of one fiber link does not disconnect the end nodes entirely.

Since a single lightpath is vulnerable to fiber link failures, it is often required to provide a high degree of reliability for this type of connection, which leads to the third configuration, i.e., a working lightpath–protection lightpath pair. In this configuration, a link-disjoint protection lightpath is precomputed and provisioned for every working lightpath [6–8]. Such protection schemes provide 100% reliability against any single-link failure in the network. However, due to various risk factors such as natural and man-caused catastrophes, disjoint fiber links may belong to the same shared-risk link group (SRLG) and fail simultaneously [9,10]. Therefore it is insufficient for the lightpaths to be merely link disjoint. Rather, it is necessary to find a working and a protection lightpath that are SRLG disjoint [11].

A drawback of SRLG-disjoint protection schemes is that if many SRLGs exist in the network it may be difficult, or even impossible, to find a working and a protection lightpath between two end nodes that are completely SRLG disjoint [12]. Thus, it may be difficult to provide 100% reliability against certain failure events. For cases in which 100% reliability is not possible, the objective should be to find one or more lightpaths for each connection, such that the reliability for each connection is maximized, or equivalently, the failure probability of the connection is minimized.

For connections of three or more end nodes, such as those in multiparty video conferencing, internet telephony, and online gaming [13–15], a common practice for achieving a high degree of reliability is establishing lightpaths between the end nodes

such that the lightpaths form a ring, which keeps the end nodes connected even when a single fiber link fails [16,17]. However, if two or more fiber links on the ring belong to the same SRLG, a failure of that SRLG may still disconnect the end nodes. Therefore it is necessary to minimize the failure probability of the lightpaths on the ring.

In this work, we introduce and study the single-lightpath problem, the parallel-lightpaths problem, and the lightpath protection problem for connections between two end nodes. We also study the lightpath-ring problem for connections of three or more end nodes. For each of the problems, we first consider the special case in which all SRLGs have uniform failure probability. We then discuss the problems in the more generic case in which the SRLGs can have different failure probabilities.

When all SRLGs have uniform failure probability, we use a unique color to distinctly represent each SRLG. If multiple links belong to the same SRLG, then all of these links are marked with the corresponding color. If a link belongs to multiple SRLGs, then that link has multiple colors. Since all SRLGs have the same failure probability, minimizing the failure probability of a lightpath is equivalent to minimizing the number of SRLGs, or colors, on the lightpath. For the single-lightpath problem, by minimizing the number of colors on the lightpath, the failure probability of the lightpath can be minimized. For the parallel-lightpaths problems, by minimizing the total number of colors on the parallel lightpaths, the probability that a failure occurs to one of the lightpaths is minimized. For the lightpath protection problem, by minimizing the number of overlapping colors on the working lightpath and the protection lightpath, the probability that a single failure event will cause both lightpaths to fail simultaneously is minimized. For the lightpath-ring problem, if we minimize the total number of colors on the lightpath ring, we minimize the failure probability of any part of the ring. On the other hand, if we minimize the overlapping colors of all the lightpaths of consecutive end nodes along the ring, we also minimize the probability of simultaneous failures of lightpaths that would disconnect the end nodes.

In this study, we evaluate the computational complexity of various minimum failure problems and prove them to be NP-hard [18]. NP-hard problems have the property that solutions that yield optimal results with polynomial time complexity have never been found [19]; thus, solutions for these problems have high computational complexity and tend to be infeasible for large networks. A problem may be found NP-hard if an existing NP-hard problem can be reduced to it using an algorithm of polynomial time complexity [19,20]. For a proven NP-hard problem, attempts can then be made to develop efficient heuristic algorithms.

Despite the seeming similarities between minimum-color lightpath problems and various versions of minimum-cost path problems, the minimum-color lightpath problems are much harder to solve. For many minimum-cost path problems, there exist efficient algorithms that solve the problems with polynomial complexity. For instance, Dijkstra's algorithm can be used to find a single minimum-cost path between two end nodes [20], and Suurballe's algorithm can be used to find two link-disjoint paths with the minimum total cost [21,22]. However, for the minimum-color lightpath problems, many of them are NP-hard, as shown later in this study. A thorough search of existing literature yields limited results in the area of interest. One relevant study looked into the problem of establishing a spanning tree using the minimum number of labels (i.e., colors) [23]. It proved the problem to be NP-hard and also proposed two heuristic algorithms. The first heuristic is named the edge replacement algorithm. The algorithm first forms an arbitrary spanning tree, then tries to replace each edge with a different edge that can reduce the total number of colors in the tree. The second heuristic is named the maximum vertex covering algorithm. This algorithm starts with an empty spanning tree, then scans through all the colors and chooses the one that covers the most uncovered vertices. This procedure is repeated until a spanning tree is formed. The studies in [24] analyzed the performance of the two heuristics and showed that the first can be arbitrarily bad while the second achieves a logarithmic approximation ratio. The work in [25] further investigated the second algorithm with slightly better approximation.

It should be pointed out that the minimum-color lightpath problems discussed in this study also apply to other areas of networking. For instance, a nationwide optical network may consist of fiber links that belong to different network carriers. A lightpath is often less expensive to establish and operate if it involves fewer carriers, even if the length of that lightpath is suboptimal. The problem of minimizing the number of

different network carriers along a lightpath thus becomes a minimum-color lightpath problem if we represent every carrier with a distinct color. Other instances of network heterogeneousness also exist and may raise similar issues that add to the relevance of the study in this work.

The rest of the paper is organized as follows. In Section 2, we study the single-lightpath problem, the parallel-lightpaths problem, and the lightpath protection problem for connections between two end nodes, as well as the lightpath-ring problems for connections of three or more end nodes. We prove these problems to be NP-hard and develop heuristic solutions for them. In Section 3, we present computer simulations and results for the heuristics. In Section 4, we conclude the paper.

## 2. Lightpath Routings for Maximum Reliability

For each of the problems in this section, we first consider the special case in which all SRLGs have uniform failure probability, and SRLGs are represented distinctly by different colors. Once the NP-hardness of the problems has been proved for this special case, we discuss the general case in which the SRLGs have different failure probabilities.

In the remaining discussions, we assume that all fiber links are bidirectional and that full wavelength-conversion capability is available at every network node; thus, the wavelength-continuity constraint is not a concern in our study. The latter assumption does not compromise the NP-hardness of the problems, since adding the wavelength-continuity constraint only increases the hardness of the problems.

### 2.A. Single-Lightpath with Minimum Failure Probability

For the special case in which all SRLGs have uniform failure probability, each SRLG is distinctly represented by a color. We refer to this instance of the single-lightpath problem as the minimum-color single-lightpath (MCSL) problem of finding a single lightpath between two end nodes such that it has the minimum number of colors. For the purpose of NP-hardness analysis, if we prove the MCSL problem to be NP-hard for networks containing only single-color links, then it will be straightforward to conclude that the MCSL problem is also NP-hard for networks possibly containing multi-color links, since the former is a special case of the latter.

The MCSL problem is defined as follows. Given network  $G=(N,L)$ , where  $N$  is the set of nodes and  $L$  is the set of fiber links, and given the set of colors  $C=\{c_1,c_2,c_3,\dots,c_K\}$ , where  $K$  is the maximum number of colors in  $G$ , and given the color  $c_l \in C$  on every link  $l \in L$ , find one lightpath from source node  $s$  to destination node  $d$  such that it uses the minimum number of colors.

#### 2.A.1. Proof of NP-hardness

We reduce a known NP-hard problem to the MCSL problem. The known NP-hard problem in this case is the minimum set covering problem [19]. This problem is stated as follows. Given a finite set  $S=\{a_1,a_2,a_3,\dots,a_n\}$ , and a collection  $C=\{C_1,C_2,\dots,C_m\}$  such that each element in  $C$  contains a subset of  $S$ , is there a minimum subset,  $C' \subseteq C$  such that every member of  $S$  belongs to at least one member of  $C'$ ?

We construct a graph  $G$  for an arbitrary instance of the minimum set covering problem, such that the graph contains one path from  $s$  to  $d$  with the minimum number of colors, if and only if  $C$  contains a minimum set cover  $C'$ . The following are the steps for the graph construction:

*Step 1.* For every element  $a_i$  in  $S$ , create a network node  $a_i$ .

*Step 2.* For every subset  $C_j$  to which  $a_i$  belongs, create a network link  $a_{i-1} a_i$  of color  $c_j$ . For element  $a_1$ , the link is  $sa_1$ . Also create a single link between  $a_n$  and  $d$  with color  $c_0$ .

An example is given in Fig. 1. In this example, we construct graph  $G$  for a minimum set covering problem  $S=\{a_1,a_2,a_3,a_4\}$ ,  $C=\{C_1,C_2,C_3,C_4C_5\}$ ,  $C_1=\{a_1,a_2\}$ ,  $C_2=\{a_2,a_3\}$ ,  $C_3=\{a_1,a_3\}$ ,  $C_4\{a_3,a_4\}$ ,  $C_5=\{a_1,a_4\}$ .

It is apparent that if there is a minimum-color path from  $s$  to  $d$ , then the colors on that path are mapped directly to a minimum set covering all the elements in  $S$ . Conversely, if there is a minimum set covering all the elements, then a minimum-color

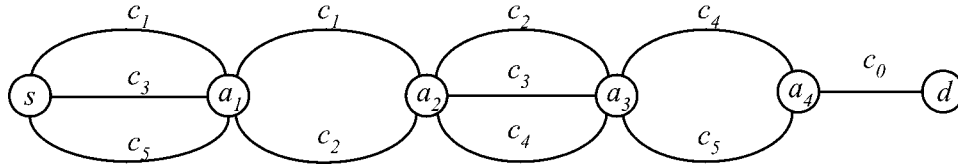


Fig. 1. Reduction of the minimum set covering problem to the MCSL problem.

path can be derived by going through every node and selecting the link with the color representing the set that covers the corresponding element.

Therefore, it is an NP-hard problem to find a single lightpath between two end nodes with the minimum number of SRLGs. Subsequently the general problem is also NP-hard where the failure probabilities of the SRLGs in the network may differ and each fiber link may belong to multiple SRLGs.

### 2.A.2. Heuristic Algorithms

We introduce two simple greedy heuristics to solve the MCSL problem. We name the first heuristic the single-lightpath color-reduction algorithm (SLCRA). The details are as follows:

*Step 1. Find a minimum-hop-count path  $p$  from  $s$  to  $d$ . Let the collection of all the colors on  $p$  be set  $C_p = \{c_1, c_2, \dots, c_k\}$ .*

*Step 2. Go through every color in  $C_p$ . Select the color such that removing all links of that color results in a minimum-hop-count path with the minimum number of colors, which is also less than  $|C_p|$ . Remove all links of the selected color from the network.*

*Step 3. Repeat steps 1 and 2 until the number of colors on the new minimum-hop-count paths cannot be further reduced.*

Step 1 of the algorithm may use one of the minimum-cost-path algorithms such as Dijkstra's algorithm by setting all link costs to 1. The running time in this step is  $O(n \log n)$  where  $n$  is the number of nodes [20]. The number of colors on this path is used as the upper bound for lightpaths, which we find in the next step. Step 2 selects one color at a time from the colors on the path obtained in step 1 (i.e.,  $C_p$ ) and tries to find a new minimum-hop-count path after temporarily eliminating all links of that particular color. After going through all the colors in  $C_p$ , we identify the color whose elimination results in the minimum number of colors on the new path. The links of that color are then permanently removed from the network graph before going to step 3. The worst-case running time of this step is  $O(mn \log n)$ , where  $m$  is the total number of colors in the network. Step 3 repeats the previous two steps until no new path can be found with fewer colors. In the worst case, the number of iterations is  $m$ , which results in the total running time of  $O(m^2n \log n)$  for this entire algorithm.

We name the next heuristic the single-lightpath all-color-optimization algorithm (SLACOA). The details are as follows:

*Step 1. Initialize link cost to 1 on all links in the network.*

*Step 2. Find a minimum-cost lightpath  $p$ . Let the number of colors on  $p$  be  $|C_p|$ .*

*Step 3. Pick one color at a time, set the link cost to zero on all links of that color, and find a new minimum-cost path. Repeat this procedure for all colors in the network, and select the color that results in the path with the minimum number of colors, which is also less than  $|C_p|$ . Keep the costs to zero on the links of that selected color.*

*Step 4. Repeat steps 2 and 3 until the number of colors on the minimum-cost paths cannot be further reduced.*

By setting all link costs to 1 in the first step, the minimum-cost path found in step 2 is effectively the minimum-hop-count path from the source  $s$  to the destination  $d$ . The running time of step 2 is  $O(n \log n)$ , where  $n$  is the number of nodes. The number of colors on this path is used as the upper bound for lightpaths that we find in the next step. Step 3 selects one color at a time from all colors in the network, sets the link cost to zero on all links of that particular color, and finds a new path between  $s$  and  $d$ . The intent is to determine whether we can reduce the number of colors on that path if we make the links of a color more attractive by setting their cost to zero. After going through all the colors, a color is identified when we find the new path with the

minimum number of colors. We then permanently set the cost of the links of that color to zero before going to step 4. The running time of this step is  $O(mn \log n)$ , where  $m$  is the total number of colors in the network. Step 4 repeats the previous two steps until no new path can be found with fewer colors. The number of iterations is  $m$ , which results in a total running time of  $O(m^2n \log n)$  for this entire algorithm.

**2.B. Parallel Lightpaths with Minimum Failure Probability**

For the special case in which all SRLGs have uniform failure probability, each SRLG is distinctly represented by a color. We refer to this instance of the parallel-lightpaths problem as the minimum total-color disjoint-lightpaths (MTCDL) problem of finding parallel lightpaths that use the minimum number of total colors. For the purpose of NP-hardness analysis, if we prove the MTCDL problem to be NP-hard for networks containing only single-color links, it will be straightforward to conclude that the MTCDL problem is also NP-hard for networks possibly containing multicolor links, since the former is a special of case of the latter.

A connection of parallel lightpaths uses at least two disjoint lightpaths. If more than two lightpaths are used, the problem becomes even harder. Here we study the two-path version of the MTCDL problem, which is defined as follows. Given network  $G=(N,L)$ , where  $N$  is the set of nodes and  $L$  is the set of fiber links, and given the set of colors  $C=\{c_1,c_2,c_3,\dots,c_K\}$ , where  $K$  is the maximum number of colors in  $G$ , and given the color  $c_l \in C$  on every link  $l \in L$ , find two disjoint lightpaths from source node  $s$  to destination node  $d$  such that the total number of colors on the two lightpaths is minimal.

*2.B.1. Proof of NP-hardness*

There are two variations of the MTCDL problem based on the requirement of light-path disjointness. In the first variation, the two lightpaths are node disjoint. In the second variation, the two lightpaths are link disjoint. We reduce the minimum set covering problem to both variations of the problem to prove their NP-hardness. For the minimum set covering problem, assume the given finite set  $S$  is  $\{a_1,a_2,a_3,\dots,a_n\}$  and the collection  $C$  is  $\{C_1,C_2,\dots,C_m\}$ .

*Variation 1. MTCDL problem with node-disjoint requirement.* We construct a graph  $G$  for an arbitrary instance of the minimum set covering problem, such that the graph contains two node-disjoint paths from  $s$  to  $d$  with the minimum total number of colors, if and only if  $C$  contains a minimum set cover  $C'$ . The following are the steps for the graph construction:

*Step 1.* For every element  $a_i$  in  $S$ , create a network node  $a_i$ .

*Step 2.* For every subset  $C_j$  to which  $a_i$  belongs, create a network link  $a_i-a_j$  of color  $c_j$ . For elements  $a_1$  and  $a_2$ , the links are  $sa_1$  and  $sa_2$ , respectively. Also create single links  $a_{n-1}d$  and  $a_nd$  with color  $c_0$ .

An example is given in Fig. 2. In this example, we construct graph  $G$  for a minimum set covering problem  $S=\{a_1,a_2,a_3,a_4\}$ ,  $C=\{C_1,C_2,C_3,C_4,C_5\}$ ,  $C_1=\{a_1,a_2\}$ ,  $C_2=\{a_2,a_3\}$ ,  $C_3=\{a_1,a_3\}$ ,  $C_4=\{a_3,a_4\}$ ,  $C_5=\{a_1,a_4\}$ .

In the constructed graph  $G$ ,  $a_{n-1}d$  and  $a_nd$  are single links. Therefore any two node-disjoint paths from  $s$  to  $d$  must have one path going along  $s-a_1-a_3-\dots-d$  and the other path going along  $s-a_2-a_4-\dots-d$ . If two node-disjoint paths  $p_1$  and  $p_2$  have the

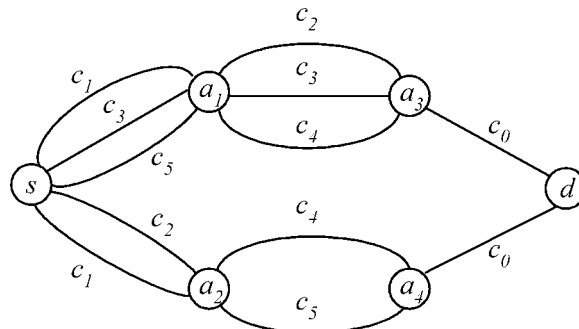


Fig. 2. Reduction of the minimum set covering problem to the MTCDL problem with node-disjoint requirement.

minimum total number of colors, since each color except for  $c_0$  is associated with a member in  $C$ , the collections of all the colors on  $p_1$  and  $p_2$  map to a minimum subset of  $C$  that covers all the elements. Conversely, if there is a minimum subset  $C' \subseteq C$  that covers all the elements, then for each node  $a_i$  in  $G$ , there is at least one member  $C_j$  in  $C'$  that contains  $a_i$ , and we choose a link  $a_{i-2}a_i$  (or  $sa_1, sa_2$ ) of the color  $c_j$ . All the links, together with the single links with color  $c_0$ , form two node-disjoint paths from  $s$  to  $d$  that have the minimum total number of colors.

*Variation 2. MTCDL problem with link-disjoint requirement.* We construct a graph  $G$  for an arbitrary instance of the minimum set covering problem, such that the graph contains two link-disjoint lightpaths from  $s$  to  $d$  with the minimum total number of colors, if and only if  $C$  contains a minimum set cover  $C'$ . The following are the steps for the graph construction:

*Step 1.* For every element  $a_i$  in  $S$ , create network nodes  $a_i$  and  $u_i$ .

*Step 2.* For every element  $a_{2i}$  in  $S$  (except for  $a_n$  if  $n$  is even, and  $a_{n-1}$  if  $n$  is odd), create a network node  $v_i$ .

*Step 3.* For every subset  $C_j$  to which  $a_i$  belongs, create a network link  $u_i a_i$  of color  $c_j$ .

*Step 4.* Create a single link  $su_1, su_2$ . If  $n$  is even, create single link  $a_1 v_1, a_2 v_1, v_1 u_3, v_1 u_4, \dots, a_{2i-1} v_i, a_{2i} v_i, v_i u_{2i+1}, v_i u_{2i+2}, \dots, a_{n-3} v_{n/2-1}, a_{n-2} v_{n/2-1}, v_{n/2-1} u_{n-1}, v_{n/2-1} u_n, a_{n-1} d, a_n d$ . If  $n$  is odd, create a single link  $a_1 v_1, a_2 v_1, v_1 u_3, v_1 u_4, \dots, a_{2i-1} v_i, a_{2i} v_i, v_i u_{2i+1}, v_i u_{2i+2}, \dots, a_{n-2} v_{(n-1)/2}, a_{n-1} v_{(n-1)/2}, v_{(n-1)/2} u_n, v_{(n-1)/2} d, a_n d$ . All of the links are of color  $c_0$ .

An example is given in Fig. 3(a). In this example, we construct graph  $G$  for a minimum set covering problem that has an even number of elements in  $S$ , i.e.,  $S = \{a_1, a_2, a_3, a_4\}$ ,  $C = \{C_1, C_2, C_3, C_4, C_5\}$ ,  $C_1 = \{a_1, a_2\}$ ,  $C_2 = \{a_2, a_3\}$ ,  $C_3 = \{a_1, a_3\}$ ,  $C_4 = \{a_3, a_4\}$ ,  $C_5 = \{a_1, a_4\}$ . Another example is given in Fig. 3(b). In this example,  $S$  has an odd number of elements, i.e.,  $S = \{a_1, a_2, a_3, a_4, a_5\}$ ,  $C = \{C_1, C_2, C_3, C_4, C_5\}$ ,  $C_1 = \{a_1, a_2, a_5\}$ ,  $C_2 = \{a_2, a_3\}$ ,  $C_3 = \{a_1, a_3, a_5\}$ ,  $C_4 = \{a_3, a_4, a_5\}$ ,  $C_5 = \{a_1, a_4\}$ .

If there are two link-disjoint paths  $p_1$  and  $p_2$  in the constructed graph  $G$ , every network node  $a_i$  must be on exactly one of the paths. If the two paths have the minimum total number of colors, since each color except for  $c_0$  is associated with a member in  $C$ , the collections of all the colors on  $p_1$  and  $p_2$  map to a minimum subset of  $C$  that covers all the elements. Conversely, if there is a minimum subset  $C' \subseteq C$  that covers all the elements, then for each node  $a_i$  in  $G$ , there is at least one member  $C_j$  in  $C'$  that contains  $a_i$ , and we choose a link  $u_i a_i$  of the color  $c_j$ . All the links, together with the single links with color  $c_0$ , form two link-disjoint paths from  $s$  to  $d$  that have the minimum total number of colors.

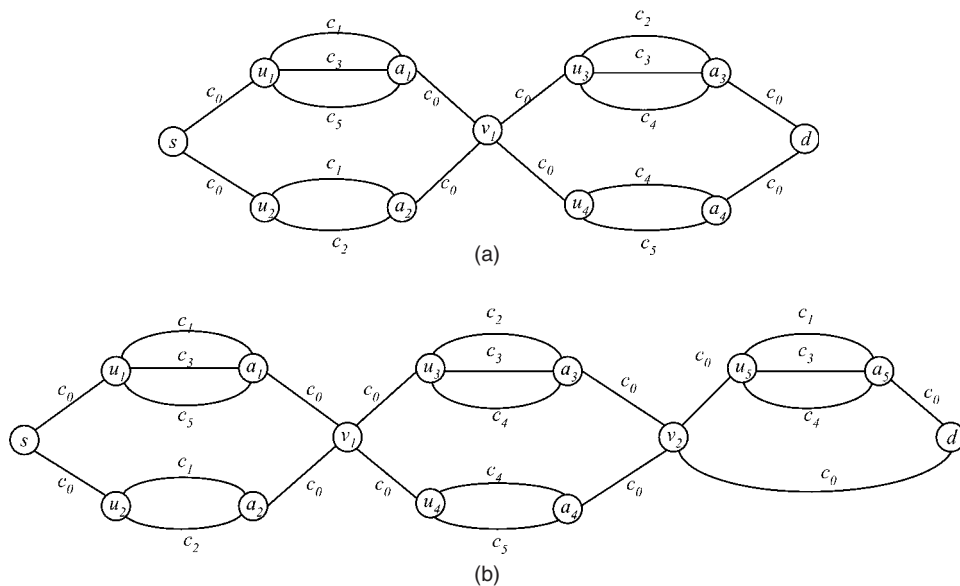


Fig. 3. Reduction of the minimum set covering problem to the MTCDL problem with link-disjoint requirement. (a) With an even number of elements. (b) With an odd number of elements.

Therefore, the problem of finding two parallel lightpaths with the minimum total number of SRLGs that are either link or node disjoint is NP-hard. Subsequently, more general versions of the problem, such as the case in which the failure probabilities of the SRLGs in the network may differ, the case in which the number of parallel lightpaths is more than two, or the case in which each fiber link may belong to multiple SRLGs, are also NP-hard.

### 2.B.2. Heuristic Algorithms

Here we introduce two simple greedy heuristics to solve the MTCDL problem with the link-disjoint requirement. Node disjointness is often not a concern due to built-in redundancy in most optical switches. We name the first heuristic the disjoint-lightpaths color-reduction algorithm (DLCRA). The details are as follows:

*Step 1. Run Suurballe's algorithm and find two link-disjoint paths  $p_1$  and  $p_2$  with the minimum-total-hop-count. Assume the collection of all the colors on  $p_1$  and  $p_2$  is set  $C_p = \{c_1, c_2, \dots, c_k\}$ .*

*Step 2. Go through every color in  $C_p$ . Select the color such that, after the links of that color are removed from the network, Suurballe's algorithm yields two link-disjoint minimum-total-hop-count paths with the minimum number of total colors, and the number of colors is also less than  $|C_p|$ . Remove the links of the selected color.*

*Step 3. Repeat steps 1 and 2 until the number of colors on the link-disjoint minimum-total-hop-count paths cannot be further reduced.*

In step 1, we run Suurballe's algorithm to find two link-disjoint lightpaths from the source node  $s$  to the destination node  $d$  with running time  $O(n^2 \log n)$  [21]. The total number of colors on the paths is used as the upper bound for lightpaths, which we find in the next step. Step 2 selects one color at a time from the colors on the two paths obtained in step 1 (i.e.,  $C_p$ ) and finds two new disjoint paths after temporarily eliminating all links of that particular color from the network graph. After repeating this procedure for all those colors, we identify the color whose elimination results in two disjoint paths with the minimum total number of colors. The links of that color are then permanently removed from the network graph before going to step 3. The worst-case running time of this step is  $O(mn^2 \log n)$ . Step 3 repeats the previous two steps until no new disjoint paths can be found with fewer total colors. In the worst case, the number of iterations is  $m$ , which results in the total running time of  $O(m^2n^2 \log n)$  for this entire algorithm.

We name the next heuristic the disjoint-lightpaths all-color-optimization algorithm (DLACOA). The details are as follows:

*Step 1. Initialize link cost to 1 on all links in the network.*

*Step 2. Run Suurballe's algorithm and find two link-disjoint paths. Assume that the total number of colors on the two paths is  $|C_p|$ .*

*Step 3. Pick one color at a time, set the link cost to zero on all links of that color; then run Suurballe's algorithm and try to find two new link-disjoint paths. Repeat this for all the colors in the network and select the one that results in two link-disjoint paths with the minimum total number of colors that is also less than  $|C_p|$ . Keep the costs to zero on the links of the selected color.*

*Step 4. Repeat steps 2 and 3 until the number of colors on the link-disjoint paths cannot be further reduced.*

By setting all link costs to 1 in the first step, the link-disjoint paths found in step 2 are effectively the minimum-total-hop-count lightpaths from the source  $s$  to the destination  $d$ . The running time of step 2 is  $O(n^2 \log n)$ , where  $n$  is the number of nodes. The total number of colors on the two paths is used as the upper bound for lightpaths, which we find in the next step. Step 3 selects one color at a time from all colors in the network and tries to find two new disjoint paths after temporarily setting the link cost to zero on all links of that particular color. The intent is to determine whether we can reduce the total number of colors on the disjoint paths if we make the links of that color more attractive by setting their cost to zero. After going through all the colors, a color is identified if the two disjoint paths we found have the minimum total number of colors. We then permanently set the cost of the links of that color to zero before going to step 4. The running time of this step is  $O(mn^2 \log n)$ . Step 4 repeats the pre-

vious two steps until no new disjoint paths can be found with fewer total colors. The number of iterations is  $m$ , which results in the total running time of  $O(m^2 n^2 \log n)$  for this entire algorithm.

### 2.C. Protected-Lightpaths with Minimum Failure Probability

For the special case in which all SRLGs have uniform failure probability, each SRLG is distinctly represented by a color. This case of the lightpath protection problem is referred to as the minimum overlapping-color disjoint-lightpaths (MOC DL) problem of finding the working lightpath and the protection lightpath that have the minimum number of overlapping colors. For the purpose of NP-hardness analysis, if we prove the MOC DL problem to be NP-hard for networks containing only single-color links, it will be straightforward to conclude that the MOC DL problem is also NP-hard for networks possibly containing multicolor links since the former is a special case of the latter.

The MOC DL problem is defined as follows. Given network  $G=(N,L)$ , where  $N$  is the set of nodes and  $L$  is the set of fiber links, and given the colors  $C=\{c_1,c_2,c_3,\dots,c_K\}$ , where  $K$  is the maximum number of colors in  $G$ , and given the color  $c_l \in C$  on every fiber link  $l \in L$ , find two link-disjoint lightpaths from source node  $s$  to destination node  $d$  such that they share the minimum number of overlapping colors.

#### 2.C.1. Proof of NP-hardness

We reduce a known NP-hard problem to the MOC DL problem. The known NP-hard problem in this case is the problem of finding two link-disjoint paths from source node  $s$  to destination node  $d$  that are completely SRLG-disjoint [12,26]. We replace every SRLG with a different color. If we were able to solve the MOC DL problem and find two link-disjoint paths from  $s$  to  $d$  with the minimum overlapping colors, then the paths should also be SRLG-disjoint if such paths exist in the network.

Therefore, the problem of finding a working lightpath and a protection lightpath with the minimum number of common SRLGs is NP-hard. Subsequently, more general versions of the problem, where the failure probabilities of the SRLGs in the network may differ or each fiber link may belong to multiple SRLGs, are also NP-hard.

#### 2.C.2. Heuristic Algorithms

We introduce three greedy heuristic algorithms for solving the MOC DL problem. We name the first heuristic the minimum-color-first-lightpath algorithm. The details are as follows:

*Step 1. Run the SLACOA from Subsection 2.A.2 and find the first lightpath  $p_1$ . Then set the link cost back to 1 on all links in the network.*

*Step 2. Increase the cost of a link if the color of the link is on  $p_1$ . The additional cost is proportional to the number of links of that color on  $p_1$ . Remove all links on  $p_1$ .*

*Step 3. Run Dijkstra's algorithm and find lightpath  $p_2$ .*

In the first step, we attempt to find a minimum-color single-lightpath from  $s$  to  $d$ . Based on the discussion in Subsection 2.A.2, the running time of this step is  $O(m^2 n \log n)$  where  $n$  is the number of nodes and  $m$  is the total number of colors in the network. In the second step, we increase the cost of all the links in the network whose colors overlap with those on the first lightpath. The more links of a particular color are on the first lightpath, the higher we increase the cost of all the links of that color. The intent is to make those links less attractive when we execute step 3. We also remove all the links on the first lightpath to ensure that the new path we find in step 3 is link disjoint from the first path. The running time of step 3 is  $O(n \log n)$ . So the total running time for the entire algorithm is  $O(m^2 n \log n)$ .

However, this algorithm may fail to find two link-disjoint lightpaths in networks containing a trap topology [27,28]. An example of such a network is depicted in Fig. 4. This network has only one wavelength. Each fiber link has a unique color. Once the first lightpath from source node  $s$  to destination node  $d$  is found along  $s-a-b-d$ , another link-disjoint lightpath cannot be found, even though two link-disjoint lightpaths do exist ( $s-e-b-d$  and  $s-a-f-d$ ). Hence we propose the second heuristic for the MOC DL problem and name it the joint-search minimum-overlapping-color algorithm, which resolves the trap topology issue. The details are as follows:



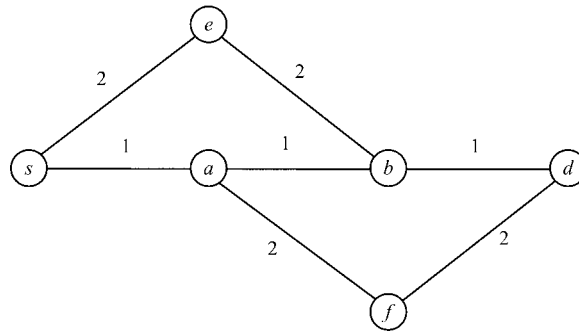


Fig. 4. Example of a trap topology. The number next to each fiber link is the link cost.

*Step 1. Set link cost to 1 on all links in the network. Then run Suurballe's algorithm and find two link-disjoint lightpaths  $p_1$  and  $p_2$ .*

*Step 2. Increase the cost of a link if the color of the link is on  $p_1$ . The additional cost is proportional to the number of links of that color on  $p_1$ .*

*Step 3. Remove all links on  $p_1$ . Run Dijkstra's algorithm and find path  $p'_1$ . Restore links on  $p_1$ . Set cost back to 1 on all links in the network.*

*Step 4. Increase the cost of a link if the color of the link is on  $p_2$ . The additional cost is proportional to the number of links of that color on  $p_2$ .*

*Step 5. Remove all links on  $p_2$ . Run Dijkstra's algorithm and find path  $p'_2$ .*

*Step 6. From the two pairs of disjoint paths  $p_1/p'_1$  and  $p_2/p'_2$ , select the pair that has fewer overlapping colors as the result.*

We start with Suurballe's algorithm in step 1. Suurballe's algorithm always finds two link-disjoint lightpaths as long as they exist. Let the two paths be  $p_1$  and  $p_2$ . The running time of this step is  $O(n^2 \log n)$ , where  $n$  is the number of nodes in the network. In steps 2 and 3, we try to find a new path  $p'_1$  that is link disjoint from  $p_1$  and is also less likely to share common colors with  $p_1$ . The running time of this step is  $O(n \log n)$ . In steps 4 and 5, we try to find a new path  $p'_2$  that is link disjoint from  $p_2$  and is also less likely to share common colors with  $p_2$ . The running time of this step is  $O(n \log n)$ . From these two pairs, we select the pair of paths that has fewer overlapping colors in step 6. The total running time for this entire algorithm is  $O(n^2 \log n)$ .

To establish an upper bound on the number of overlapping colors for the results obtained from the two previous heuristics, we develop a simple two-step algorithm. Details of this algorithm are as follows:

*Step 1. Set link cost to 1 on all links in the network. Run Dijkstra's algorithm and find the shortest lightpath  $p_1$ .*

*Step 2. Increase the cost of a link if the color of the link is on  $p_1$ . The additional cost is proportional to the number of links of that color on  $p_1$ .*

*Step 3. Remove all links on  $p_1$ . Run Dijkstra's algorithm again and find the second shortest path  $p_2$ .*

Paths  $p_1$  and  $p_2$  are link disjoint. Since the costs on the links of those colors on  $p_1$  are increased, it is expected that  $p_1$  and  $p_2$  share fewer colors. The running time of this heuristic is the same as that of Dijkstra's algorithm, which is  $O(n \log n)$ .

## 2.D. Lightpath Ring with Minimum Failure Probabilities

For the special case in which all SRLGs have uniform failure probability, the lightpath-ring reliability problem becomes the minimum total-color lightpath-ring (MTCLR) problem of finding a lightpath ring that has the minimum number of total colors on all the constituent lightpaths, and the minimum overlapping-color lightpath-ring (MOCLR) problem of finding a lightpath ring that has the minimum number of overlapping colors on all the constituent lightpaths. For the purpose of NP-hardness analysis, if we prove the MTCLR problem and MOCLR problem NP-hard for networks containing only single-color links, it will be straightforward to conclude that these problems are also NP-hard for networks possibly containing multicolor links, since the former are special cases of the latter.

### 2.D.1. Lightpath Ring with Minimum Failure Probability on Any Constituent Lightpaths

The MTCLR problem is defined as follows. Given network  $G=(N,L)$ , where  $N$  is the set of nodes and  $L$  is the set of fiber links, and given the colors  $C=\{c_1,c_2,c_3,\dots,c_K\}$ , where  $K$  is the maximum number of colors in  $G$ , and given the color  $c_l \in C$  on every fiber link  $l \in L$ , find the lightpaths that connect node  $n_1,n_2,\dots,n_m$  such that the total number of colors on the lightpaths is minimized.

This problem is reducible to the Hamiltonian-cycle problem [19], which is a known NP-hard problem, by the following steps:

*Step 1. For the network graph  $G$  of a Hamiltonian-cycle problem, construct an identical network graph  $G'$ .*

*Step 2. Assign a unique color to every link in  $G'$ .*

Hence, a minimum-color ring connecting all nodes in  $G'$  is reduced to a Hamiltonian-cycle in  $G$  and this problem is proven NP-hard. Therefore, the problem of finding a lightpath ring with the minimum number of total SRLGs when the SRLGs have uniform failure probability is NP-hard. Subsequently the general problem is also NP-hard where the failure probabilities of the SRLGs in the network may differ and each fiber link may belong to multiple SRLGs. Due to the page limit, we have omitted the heuristics for this problem. They will be studied in a separate paper.

### 2.D.2. Lightpath Ring with Minimum Probability of Simultaneous Failures of Multiple Constituent Lightpaths

The MOCLR problem is defined as follows. Given network  $G=(N,L)$ , where  $N$  is the set of nodes and  $L$  is the set of fiber links, and given the colors  $C=\{c_1,c_2,c_3,\dots,c_K\}$ , where  $K$  is the maximum number of colors in  $G$ , and given the color  $c_l \in C$  on every fiber link  $l \in L$ , find the lightpaths that connect node  $n_1,n_2,\dots,n_m$  (can be in any order) such that the number of overlapping colors on the lightpaths is minimized.

This problem is an NP-hard problem because it contains the MOCDL problem of Subsection 2.C as a special case when  $m=2$ . Subsequently the general problem is also NP-hard where the failure probabilities of the SRLGs in the network may differ and each fiber link may belong to multiple SRLGs. Due to the page limit, we have omitted the heuristics for this problem. They will be studied in a separate paper.

To summarize Section 2, we discussed five lightpath reliability problems with the objectives of minimizing the failure probability of the connections. For each of the problems, it was first shown that the problem is NP-hard for the special case in which all SRLGs have uniform failure probability and each fiber link belongs to a single SRLG. Then it becomes evident that the general problems are also NP-hard when the SRLGs have different failure probabilities and each fiber link may belong to multiple SRLGs. Heuristic solutions were developed for the first three problems.

## 3. Computer Simulations

We implement computer simulations to evaluate the heuristics on networks that are randomly generated using LEDA [29]. The network size ranges from 10 to 40 nodes. The nodal degree ranges from 2.6 to 3.0. Each fiber link is assumed to support an unlimited number of lightpaths and is assumed to belong to any SRLG with equal probability. The color intensity of the network ranges from 1 to 20. The network color intensity is defined as the average number of links of the same color. When the color intensity is 1, every network link has a different color and therefore belongs to a different SRLG. When the color intensity increases, more links have the same color, i.e., the links belong to the same SRLG. Subsequently more fiber links may fail simultaneously. If the color intensity is equal to the number of fiber links in a network, then all links have the same color and they always fail simultaneously. In addition to the heuristics, we also developed integer linear programming (ILP) formulations (Appendix A) in order to obtain the optimal solutions using CPLEX [30]. We observed that the difference between the amount of time required for the execution of the heuristics and the amount of time required for solving the ILPs was significant. For example, using an Intel-based personal computer, it took only a few minutes to execute each of the

heuristics for 10,000 end-node pairs on a 40-node network, whereas it took several hours to solve the ILPs for the same end-node pairs and network.

### 3.A. Simulations for the Minimum-Color Single-Lightpath Problem

We developed two heuristics for the MCSL problem, i.e., the SLCRA and the SLACOA. For every randomly chosen pair of end nodes, we solved the ILP and used the heuristics to obtain the minimum-color lightpath. We then compared the average numbers of colors on all the lightpaths. To establish an upper bound for the results of the heuristics, we also ran Dijkstra's algorithm to obtain the minimum-hop-count paths for all the end-node pairs. Two sets of the results are depicted in Figs. 5 and 6. Results on other network topologies are similar.

We note that, as the nodal degree increases, the number of colors on the lightpath decreases. The reason for this behavior is that an increase in nodal degree results in a wider choice of available routes for each connection. Furthermore, an increase in nodal degree reduces the average hop distance for each connection, thereby reducing the number of colors. We also note that color intensity has an impact on the number of colors on the lightpaths as well. When the color intensity is 1, all fiber links in the network have a distinct color; hence, the number of colors for a given lightpath is simply the hop distance of that lightpath, and the average number of colors for each lightpath is simply the average hop distance in the network. As the color intensity increases, the number of links with the same colors also increases. As a result, the number of unique colors on the lightpaths decreases. The network topology and the size of the network also affect the number of colors on the lightpath. Larger networks with more nodes result in a higher average hop count for lightpaths; hence, for the same nodal degree and color intensity, lightpaths in a network with more nodes will have a greater number of colors than lightpaths in a network with fewer nodes.

### 3.B. Simulations for the Parallel-Lightpaths Problem

We developed two heuristics for the MTCDL problem, i.e., the DLCRA and the DLA-COA. For every randomly chosen pair of end nodes, we solved the ILP and used the heuristics to obtain two parallel lightpaths with a minimum total number of colors. We then compared the average total numbers of colors on the lightpaths. To establish

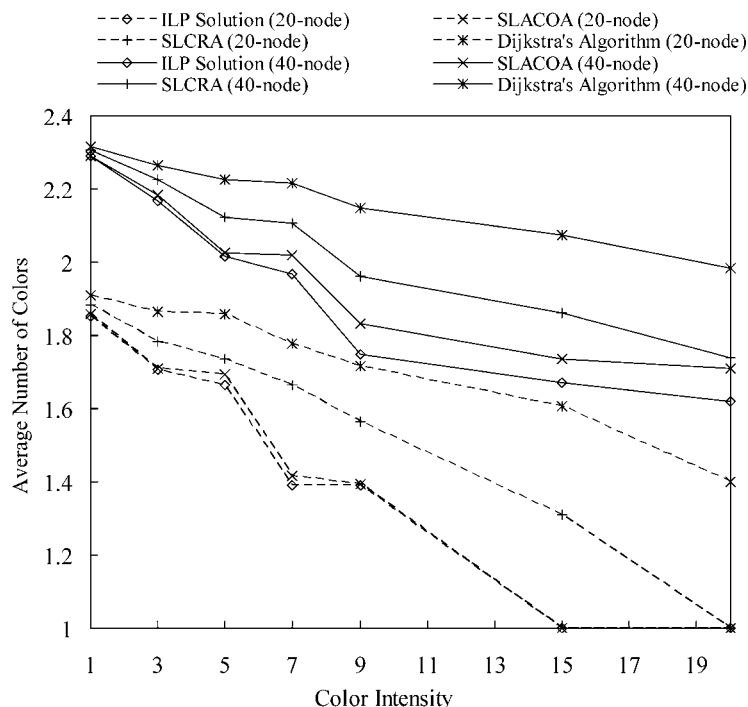


Fig. 5. Average number of colors on lightpaths of all source-destination pairs versus network color intensity. Network nodal degree=2.6.

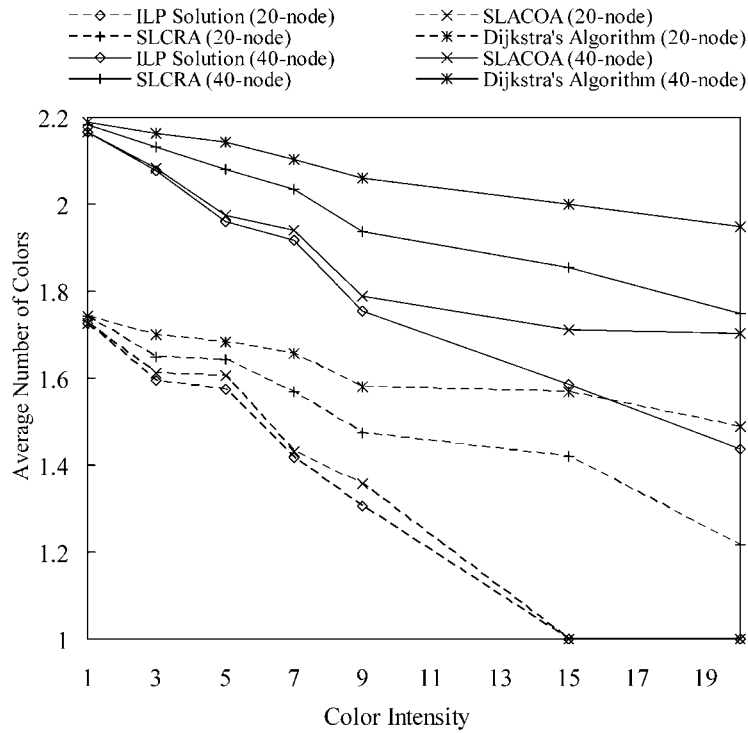


Fig. 6. Average number of colors on lightpaths of all source-destination pairs versus network color intensity. Network nodal degree=3.0.

an upper bound for the results of the heuristics, we ran Suurballe's algorithm on all the end-node pairs. Two sets of the results are depicted in Figs. 7 and 8. Results on other network topologies are similar.

Based on the simulation results, the parallel lightpaths obtained from DLACOA are closest to the optimal ILP solutions in the average number of total colors on the lightpaths. This is because DLACOA selects optimal colors from all colors in the network while DLCRA is restricted to the colors on the two initial lightpaths.

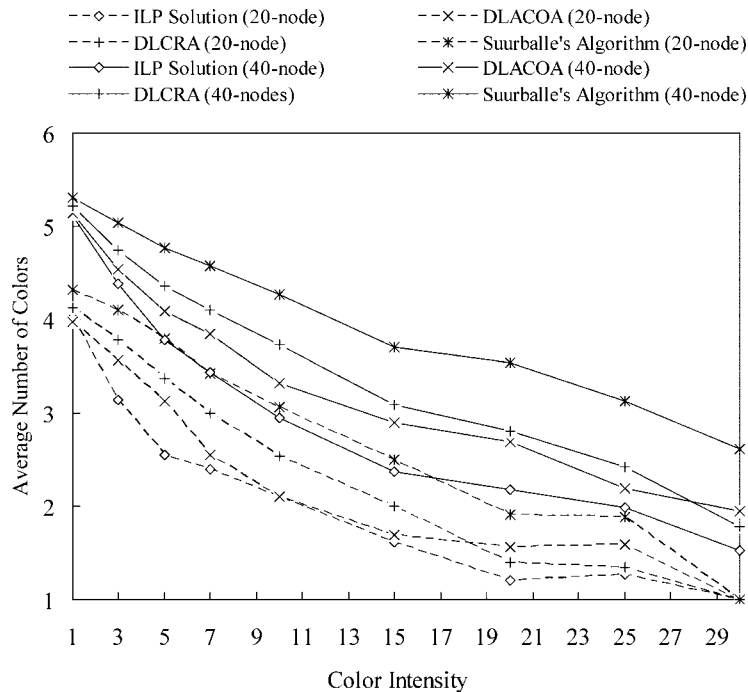


Fig. 7. Average total number of colors on two parallel lightpaths of all source-destination pairs versus network color intensity. Network nodal degree=2.6.

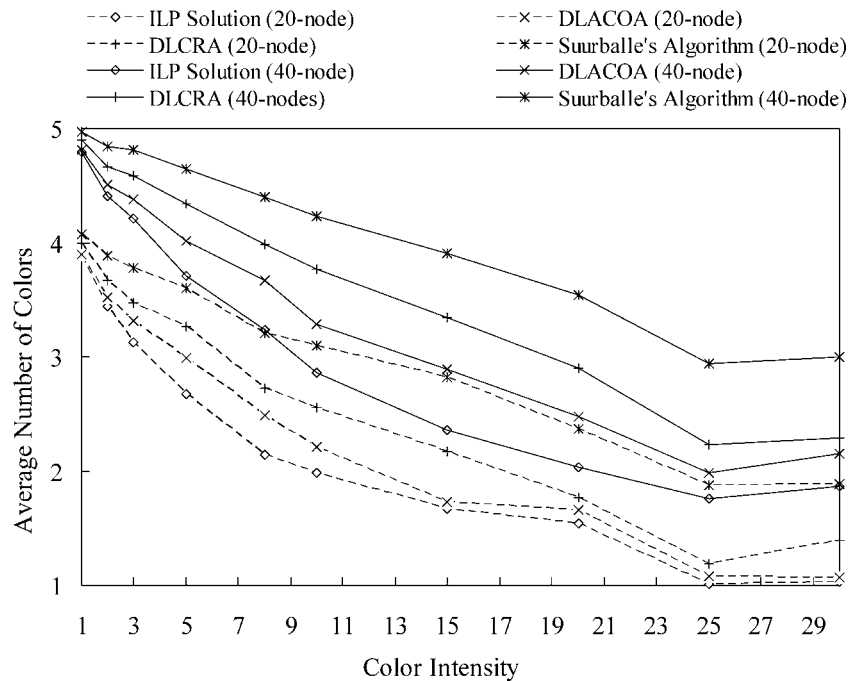


Fig. 8. Average total number of colors on two parallel lightpaths of all source–destination pairs versus network color intensity. Network nodal degree=3.0.

The network nodal degree, the color intensity, and the network size have similar impacts on the number of total colors on the parallel lightpaths as in the simulations in Subsection 3.A. Higher nodal degree, or greater color intensity, or smaller network size all contribute to fewer total colors on the parallel lightpaths.

### 3.C. Simulations for the Protected-Lightpaths Problem

We developed two heuristics for the MOC DL problem, i.e., the minimum-color-first-lightpath algorithm (MCFLA) and the joint-search minimum-overlapping-color algorithm (JSMOCA). We used the simple two-step algorithm (STSA) to establish an upper bound on the number of overlapping colors for the two heuristics.

For every randomly chosen pair of end nodes, we solved the ILP and used the heuristics to obtain working and protection lightpaths with a minimum number of overlapping colors. We then compared the average numbers of overlapping colors on all the lightpath pairs. Two sets of the results are depicted in Figs. 9 and 10. Results on other network topologies are similar.

Based on the simulation results, the protected lightpaths obtained from the JSMOCA are closest to the optimal ILP solutions in the average numbers of overlapping colors on the lightpaths. This is because JSMOCA selects optimal colors from all colors in the network while MCFLA is restricted to the colors on the initial pair of lightpaths.

The number of overlapping colors is directly related to how susceptible the two lightpaths are to a single SRLG failure event. As the color intensity increases, more fiber links belong to the same SRLG and it is more likely that the two lightpaths belong to a greater number of common SRLGs. Thus, the lightpaths become more susceptible to simultaneous failures.

## 4. Conclusions

In this paper we have discussed five lightpath reliability problems. For connections between two end nodes, we discussed the single-lightpath problem, the parallel-lightpaths problem, and the protected-lightpaths problem. For the interconnection of three or more end nodes, we discussed the lightpath-ring problems. When all SRLGs have uniform failure probability, these problems are formulated as minimum-color lightpath problems (MCSL, MTC DL, and MOC DL) or minimum-color lightpath-ring problems (MTCLR and MOCLR). We proved these problems NP-hard and extended

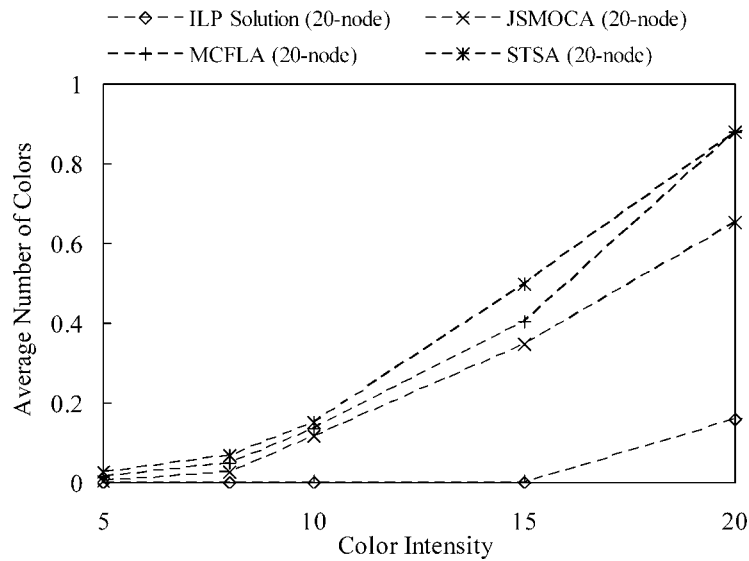


Fig. 9. Average number of overlapping colors on protected lightpaths of all source-destination pairs versus network color intensity on a 20-node network. Nodal degree = 2.6.

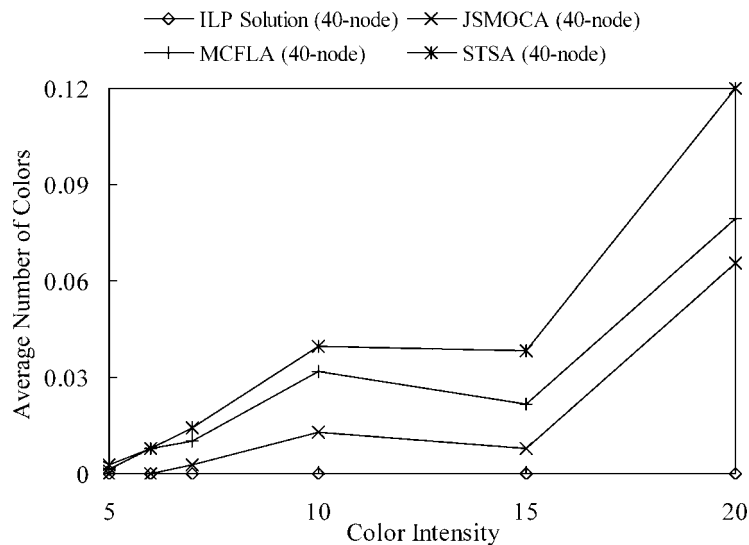


Fig. 10. Average number of overlapping colors on protected lightpaths of all source-destination pairs versus network color intensity on a 40-node network. Nodal degree = 3.0.

the conclusions to the general problems where the SRLGs may have different failure probabilities and each fiber link may belong to multiple SRLGs. We proposed various heuristics that execute in polynomial time and that may be suitable for large-scale networks. Despite the simplicity of the heuristics, computer simulations demonstrated that the heuristics yield solutions that are close to optimal.

From the simulations, we also observed that various factors affect the number of colors (i.e., SRLGs) on the lightpaths, including the nodal degree, the color intensity, and the number of nodes in the network. An increase in the nodal degree helps reduce the number of colors on the lightpaths for the problems that attempt to minimize the total number of colors on the lightpaths (MCSL, MTCDL). This is due to the fact that there is a greater choice of routes for the lightpaths. The heuristic algorithm SLACOA performs the best for reducing the total number of colors on a single lightpath (MCSL), whereas DLACOA is the most successful with the parallel-lightpaths problem (MTCDL).

The color intensity also affects the number of colors on the lightpaths. An increase in the color intensity increases the number of overlapping colors between a working

lightpath and its protection lightpath, thereby making both lightpaths more susceptible to simultaneous failures (MOCDL). JSMOCA reduces the number of overlapping colors and is reasonably close to the ILP's optimal solution.

While the emphasis of this paper is to identify the problems and to prove their NP-hardness, future work may involve the development of approximation algorithms for each of the problems. For instance, the minimum set covering problem is reducible to the MCSL problem with polynomial complexity as shown in Subsection 2.A. Based on the significant number of studies already done on the approximation algorithms for the minimum set covering problem [19,31], similar work can be extended for the MCSL problem.

Another interesting topic for future work would be to consider the multiple-lightpath problems (MTCDL and MOCDL) without the link-disjoint constraint. In this case, the lightpaths may share a common fiber link if the failure probability of that link is low.

## Appendix A

All ILP formulations developed below are for static traffic. The problem with a single connection is a special case when there is only one source–destination pair in  $\Delta$ .

### 1. ILP Formulation for the Minimum-Color Single-Lightpath Problem

The following are given as inputs to the problem:

- $N$ : number of nodes in the network.
- $L$ : number of links in the network.
- $\text{color}_c^{ij}$ : 1 if link  $ij$  has color  $c$ ; 0 otherwise.
- $\Delta = \{s_1d_1, s_2d_2, \dots, s_md_m, \dots, s_Md_M\}$ ,  $M \geq m \geq 1$ : All the source–destination pairs of the connection requests.

The ILP solves for the following variables:

- $\alpha_m^{ij}$ : 1 if link  $ij$  is used on the lightpath between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\delta_m^c$ : 1 if color  $c$  is on the lightpath between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.

Objective: minimize the average number of colors on the lightpaths,

$$\left( \sum_m \sum_c \delta_m^c \right) / M. \quad (\text{A1})$$

Constraints: Eqs. (A2)–(A4) describe the flow constraints, i.e., each lightpath goes through a fiber link at most once:

$$\sum_j \alpha_m^{xj} = 1, \quad \text{where } x = s_m, \quad \forall m, \quad (\text{A2})$$

$$\sum_i \alpha_m^{ik} - \sum_j \alpha_m^{kj} = 0, \quad \forall k \neq s_m, d_m, \quad \forall m, \quad (\text{A3})$$

$$\sum_j \alpha_m^{jy} = 1, \quad \text{where } y = d_m, \quad \forall m. \quad (\text{A4})$$

Inequalities (A5) and (A6) describe the upper and lower bounds of the number of occurrences of a particular color on a lightpath:

$$\delta_m^c \leq \sum_{i,j} (\text{color}_c^{ij} \times \alpha_m^{ij}), \quad \forall m \forall c, \quad (\text{A5})$$

$$L \delta_m^c \geq \sum_{i,j} (\text{color}_c^{ij} \times \alpha_m^{ij}), \quad \forall m \forall c. \quad (\text{A6})$$

## 2. ILP Formulation for the Parallel-Lightpaths Problem

The following are given as inputs to the problem:

- $N$ : number of nodes in the network.
- $L$ : number of links in the network.
- $\text{color}_c^{ij}$ : 1 if link  $ij$  is of color  $c$ ; 0 otherwise.
- $\Delta = \{s_1d_1, s_2d_2, \dots, s_md_m, \dots, s_Md_M\}$ ,  $M \geq m \geq 1$ : All the source–destination pairs of the connection requests.

The ILP solves for the following variables:

- $\alpha_m^{ij}$ : 1 if link  $ij$  is used on lightpath  $p_1$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\beta_m^{ij}$ : 1 if link  $ij$  is used on lightpath  $p_2$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\delta_{1m}^c$ : 1 if color  $c$  is on lightpath  $p_1$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\delta_{2m}^c$ : 1 if color  $c$  is on lightpath  $p_2$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\text{overlap}_m^c$ : 1 if color  $c$  is on both lightpaths  $p_1$  and  $p_2$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.

Objective: minimize the average total number of colors on the parallel-lightpaths,

$$\left( \sum_m \sum_c (\delta_{1m}^c + \delta_{2m}^c - \text{overlap}_m^c) \right) / M. \quad (\text{A7})$$

Constraints: Eqs. (A8)–(A13) describe the flow constraints for the two parallel lightpaths, i.e., each lightpath goes through a fiber link at most once:

$$\sum_j \alpha_m^{xj} = 1, \quad \text{where } x = s_m, \quad \forall m, \quad (\text{A8})$$

$$\sum_i \alpha_m^{ik} - \sum_j \alpha_m^{kj} = 0, \quad \forall k \neq s_m, d_m, \quad \forall m, \quad (\text{A9})$$

$$\sum_j \alpha_m^{jy} = 1, \quad \text{where } y = d_m, \quad \forall m, \quad (\text{A10})$$

$$\sum_j \beta_m^{xj} = 1, \quad \text{where } x = s_m, \quad \forall m, \quad (\text{A11})$$

$$\sum_i \beta_m^{ik} - \sum_j \beta_m^{kj} = 0, \quad \forall k \neq s_m, d_m, \quad \forall m, \quad (\text{A12})$$

$$\sum_j \beta_m^{jy} = 1, \quad \text{where } y = d_m, \quad \forall m. \quad (\text{A13})$$

Inequalities (A14)–(A17) describe the upper and lower bounds of the number of occurrences of a particular color on each of the two lightpaths:

$$\delta_{1m}^c \leq \sum_{ij} (\text{color}_c^{ij} \times \alpha_m^{ij}), \quad \forall m \forall c, \quad (\text{A14})$$

$$L \delta_{1m}^c \geq \sum_{ij} (\text{color}_c^{ij} \times \alpha_m^{ij}), \quad \forall m \forall c, \quad (\text{A15})$$

$$\delta_{2m}^c \leq \sum_{ij} (\text{color}_c^{ij} \times \beta_m^{ij}), \quad \forall m \forall c, \quad (\text{A16})$$

$$L \delta_{2m}^c \geq \sum_{ij} (\text{color}_c^{ij} \times \beta_m^{ij}), \quad \forall m \forall c. \quad (\text{A17})$$



Inequality (A18) is the overlapping-color constraint that determines the set of overlapping colors of the parallel lightpaths, i.e.,  $\text{overlap}_m^c = 1$  iff  $\delta_{1m}^c = \delta_{2m}^c = 1$  and  $\text{overlap}_m^c = 0$  iff  $\delta_{1m}^c \neq \delta_{2m}^c$ ,

$$0 \leq 1 - \delta_{1m}^c + 1 - \delta_{2m}^c + 2\text{overlap}_m^c \leq 2, \quad \forall m \forall c. \quad (\text{A18})$$

Inequalities (A19) and (A20) are the link-disjoint constraint for the two lightpaths:

$$\alpha_m^{ij} + \beta_m^{ij} \leq 1, \quad \forall i, j, \quad \forall m, \quad (\text{A19})$$

$$\alpha_m^{ji} + \beta_m^{ji} \leq 1, \quad \forall i, j, \quad \forall m. \quad (\text{A20})$$

### 3. ILP Formulation for the Protected-Lightpaths Problem

The following are given as inputs to the problem:

- $N$ : number of nodes in the network.
- $L$ : number of links in the network.
- $\text{color}_c^{ij}$ : 1 if link  $ij$  is of color  $c$ ; 0 otherwise.
- $\Delta = \{s_1d_1, s_2d_2, \dots, s_md_m, \dots, s_Md_M\}$ ,  $M \geq m \geq 1$ : all the source–destination pairs of the connection requests.

The ILP solves for the following variables:

- $\alpha_m^{ij}$ : 1 if link  $ij$  is used on lightpath  $p_1$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\beta_m^{ij}$ : 1 if link  $ij$  is used on lightpath  $p_2$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\delta_{1m}^c$ : 1 if color  $c$  is on lightpath  $p_1$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\delta_{2m}^c$ : 1 if color  $c$  is on lightpath  $p_2$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.
- $\text{overlap}_m^c$ : 1 if color  $c$  is on both  $p_1$  and  $p_2$  between source–destination pair  $s_md_m$  in  $\Delta$ ; 0 otherwise.

Objective: minimize the average number of overlapping colors on the working-protection-lightpaths pairs,

$$\left( \sum_m \sum_c \text{overlap}_m^c \right) / M. \quad (\text{A21})$$

Constraints: same as those in Eqs. (A8)–(A13) and inequalities (A14)–(A20).

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