Heuristic algorithms for finding reliable lightpath under multiple failures[1]

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ABSTRACT

In this work, we study the NP-hard problem of maximizing lightpath reliability in optical mesh networks against simultaneous failures of multiple fiber links. The fiber links belong to shared risk link groups (SRLGs) that have arbitrary failure probabilities. We study three heuristic algorithms and evaluate their performance through simulations.

1. INTRODUCTION

A lightpath in WDM networks offers data rate up to 40 Gbits/s or even 100Gbits/s, which makes it an ideal transmission medium for the next generation transport networks [1][2][3][4]. With such high data rates, a failure of a lightpath can potentially cause significant service disruption to the customers. Therefore it is among the top priorities of network operators to ensure lightpath reliability.

At the expense of resource utilization and network management, one may achieve high lightpath reliability through protection in which a link-disjoint protection lightpath is pre-computed and reserved for each working lightpath [5][6][7]. Such schemes provide 100% reliability against single-link failures. However, various risk factors such as natural and man-caused catastrophes introduce the possibility that when a network failure occurs, multiple fiber links that belong to the same shared risk link group (SRLG) fail simultaneously [8][9][10][11]. In this case, only a working and a protection lightpath that are SRLG-disjoint can survive the failure [12]. However, the study in [13] proved that it is NP-hard to find SRLG-disjoint working and protection lightpaths. Consequently, attempts were made to minimize the probability of simultaneous failures of the pair. Yuan et al. proved this problem also NP-hard and proposed heuristic algorithms for the special cases in which all SRLGs have equal failure probabilities [14][15]. Various heuristic solutions were also proposed for the general case in which the SRLGs have different failure probabilities [16][17][18].

Shared path protection may be used to improve resource utilization but it further increases the complexity of signaling and network management [18][19]. Therefore for certain types of traffic and customers, it may be more resource-efficient and cost-effective to use a single lightpath without protection, if the reliability of the lightpath can be maximized.

As later shown in Section 2, for networks with only single failures, finding the maximum reliable lightpath is equivalent to a minimum-cost-path problem and can be easily solved. However, simultaneous multiple failures complicate the matter and make the task NP-hard [14][15]. In this paper, we propose three heuristic algorithms to solve the problem.

The remainder of the paper is organized as follows. In Section 2, we describe the maximum reliable lightpath problem. In Section 3, we propose three heuristic solutions. The algorithms are evaluated through computer simulations in Section 4. We conclude the paper in Section 5.

2. PROBLEM DESCRIPTION

The problem is defined as follows. Given network $G = (N, L, S, P_s)$ where N is the set of nodes, L is the set of fiber links (assume they are bidirectional), and S is the set of SRLGs in the network, $P_s = \{p_1, p_2, p_3, \ldots, p_i, \ldots p_{|S|}\}$ is the set of non-failure probabilities of each SRLG $s_i \in S$ and $0 < p_i < 1$, also given $S^l\ \forall\ l \in L$, and $S^l \subseteq S$ is the set of SRLGs to which link l belongs, find one lightpath P from source node s to destination node t such that P has the maximum reliability.

The reliability of a lightpath is defined as follows. Let S^P be the set of all SRLGs to which the links of a lightpath P belong. Then the reliability of P is

$$r^P = \prod_{s_i^P \in S^P} p_i\ ,\ p_i \in P_s \text{ and } p_i \text{ is the non-failure probability of SRLG } s_i^P \qquad \text{(Eq. 1)}$$

In this study, we are only considering fiber link failures and not node failures because nodal devices in modern transport networks often have built-in redundancies and are

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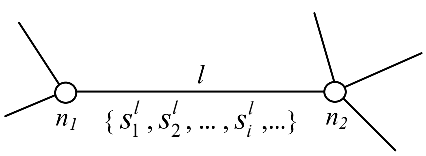
located in well-controlled facilities, which make them very reliable.

A single SRLG may contain multiple fiber links while a fiber link may belong to multiple SRLGs. However, we can transform a link belonging to multiple SRLGs into multiple links each belonging to a single SRLG without altering the reliabilities of the lightpaths in the network. This is explained as follows.

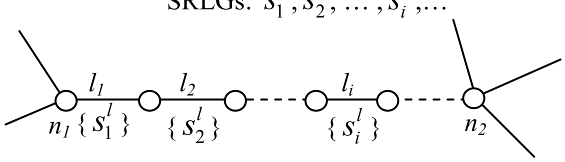
Let the set of SRLGs to which l belongs be $S^l = \{ s_1^l, s_2^l, \ldots, s_i^l, \ldots \}$ and $|S^l| > 1$. The non-failure probability of l is

$$r^l = \prod_{s_i^l \in S^l} p_i\ ,\quad p_i \in P_s\ , \text{ and } p_i \text{ is the non-failure probability of SRLG } s_i^l \qquad \text{(Eq. 2)}$$

Now replace l with $|S^l|$ concatenating links $l_1, l_2, \ldots, l_i, \ldots$, each belonging to a unique SRLG $s_1^l, s_2^l, \ldots, s_i^l, \ldots$ from S^l, as shown in Fig. 1. The reliability of l_i is the non-failure probability of s_i^l, i.e., p_i. Thus the reliability of the combined links $l_1, l_2, \ldots, l_i, \ldots$ is $p_1 \times p_2 \times \ldots \times p_i \times \ldots$, which is the same as r^l. Also because there is no other links branching off from these links, a lightpath going through one of the links must also go through all of them. Therefore their effects on the reliabilities of lightpaths are the same as that of l. Consequently, without the loss of generality, we assume for the remainder of the paper, all fiber links each belongs to a single SRLG, i.e., $|S^l| = 1\ \forall\ l \in L$.



a. Link l between nodes n_1 and n_2 belonging to multiple SRLGs: $s_1^l, s_2^l, \ldots, s_i^l, \ldots$



b. Multiple concatenating links each belonging to a single SRLGs: l_1 to s_1^l, l_2 to s_2^l, … , l_i to s_i^l, …

Fig. 1. Transforming a fiber link that belongs to multiple SRLGs into multiple links each belonging to a single SRLG.

3. Analysis and Solutions

In the special case in which every fiber link belongs to a unique SRLG, the maximum reliable lightpath can be readily obtained using the following algorithm:

Algorithm (A-1)	
Step 1:	*for (every link $l_i \in L$) {* *set its link cost to $c_i = |\log p_i|$ where p_i is the non-failure probability of SRLG s_i^l to which l_i belongs;* *}*
Step 2:	*run Dijkstra's algorithm for a minimum-cost path between node s and t; [20]* *if (succeeds) return the path;* *else print "no path exists";*

The running time of this algorithm is the time of computing the cost of every link plus the minimum-cost-path algorithm, i.e., $O(|L|+|N|\log|N|)$ [20]. Its correctness can be proven as follows.

Proof: Let P be a lightpath connection node s and t. Let the collection of links of P be $L^P = \{ l_1^P, l_2^P, \ldots, l_i^P, \ldots \}$. From Step 1, the link cost c_i of link l_i^P is $|\log p_i|$ where p_i is the non-failure probability of the SRLG s_i^l to which l_i^P belongs, and $0 < p_i < 1$.

Thus the total cost of lightpath P is

$$c^P = \sum_{l_i^P \in L^P} c_i = \sum_{l_i^P \in L^P} |\log p_i| = |\log(\prod_{l_i^P \in L^P} p_i)|$$
$$= \log (\prod_{l_i^P \in L^P} p_i)^{-1}, \text{ and } 1 < (\prod_{l_i^P \in L^P} p_i)^{-1} < +\infty$$

Hence, $\prod_{l_i^P \in L^P} p_i$ is maximized when c^P is minimized.

Since every link belongs to a unique SRLG, the reliability of P is $r^P = \prod_{l_i^P \in L^P} p_i$, $0 < r^P < 1$. Thus when c^P is minimized, r^P is maximized. □

This problem becomes NP-hard if each SRLG contains multiple fiber links that may fail simultaneously [14][15]. For networks in which the number of SRLGs is small, one may execute (A-1) for all subnetworks of G, each containing only the links that belong to a subset of SRLGs in S, and choose the lightpath with the maximum reliability computed using Eq.1. The number of subnetworks is $2^{|S|}-1$. For networks with arbitrarily large number of SRLGs, we propose three heuristic solutions.

3.1. Heuristic solution (HA-1)

Heuristic solution (HA-1) is a modified Dijkstra's

Algorithm. In this algorithm, during the process of generating the preferred lightpath P from node s to t, let S^P the set of SRLGs to which the known portion of P belongs, if the SRLG s_i^l of a fiber link l_i is already in S^P, we consider l_i's link cost to be zero because adding l_i to P does not decrease P's reliability. In this way, new links may be added to P without introducing new SRLG to S^P and lowering its reliabilities. The details are as follows.

Algorithm (HA-1)

```
Step 1:  same as Step 1 of (A-1);
Step 2:  initialize set ND to contain all nodes except s;
Step 3:  initialize array CT[] so that CT[v] is the cost of
         the link from the source to v if v is adjacent to s,
         and INFINITY otherwise;
Step 4:  initialize entries of array PR[] so that PR[v] is
         assigned s if v is adjacent to s; NULL otherwise;
Step 5:  initialize array S[] so that S[v] contains the set of
         SRLGs to which the preferred path from s to v
         belongs to. Initially S[v] is ∅ ∀ v∈ N;
Step 6:  while (ND ≠ ∅){
            choose a node u from ND such that CT[u] is
            minimum;
            if (CT[u] is INFINITY)
               output "no path exists" and quit;
            delete u from ND;
            S[u]=S[PR[u]] ∪ {SRLG of link <PR[u], u>};
            if (u is t) {
               generate the preferred path from PR[] as
               s - ... - PR[PR[u]] - PR[u] - u;
               quit;
            }
            for (each node v that is adjacent to u){
               if (v ∈ ND){
                  if (the SRLG of link <u, v>∈ S[u])
                     c = CT[u];
                  else
                     c = CT[u] + cost of link <u, v>;
                  if (c < CT[v]){
                     PR[v] = u;
                     CT[v] = c;
                  }
               }
            }
         }
```

This algorithm has the same running time as Dijkstra's algorithm which is $O(|N|log|N|)$.

3.2. Heuristic solution (HA-2)

The maximum reliable lightpath problem is reducible to the NP-hard minimum set-covering problem [14]. Chvatal studied an $O(log|N|)$-approximation algorithm for the set-covering problem [21]. Here we modify Chavatal's algorithm and propose heuristic solution (HA-2). The details are as follows:

Algorithm (HA-2)

```
Definition:  let EN(Si) be the set of unique end-nodes of the
             links that belong to SRLG Si ;
             let |EN(Si)| be the size of EN(Si);
Step 1:      set J = ∅ ;
Step 2:      if (EN(Si) = ∅ for all Si∈ S – J)
                print "No path exist" and quit;
Step 3:      find a SRLG with the maximum ratio
             |EN(Si)|/pi ;
Step 4:      add Si to J;
             remove EN(Si) from EN(Sj) for all Sj ∈ S - Si ;
Step 5:      run (A-1) for subnetwork of G with only the
             links that belong to the SRLGs in J;
             if (succeeds)  return the path;
             else  goto Step 2;
```

Ratio $|EN(S_i)|/p_i$ counts the number of end-nodes of the links that belong to SRLG S_i per unit of non-failure probability. Intuitively, if a lightpath consists of the links that belong to the SRLGs with greater $|EN(S_i)|/p_i$ ratios, the number of SRLGs which the lightpath is associated with is likely to be small. Hence the reliability of the lightpath is likely to be greater. The running time of this algorithm is $O(|S|(|L|+|N|\log|N|))$.

3.3. Heuristic solution (HA-3)

From Eq. 1, if the non-failure probabilities of the SRLGs are close to each other, then the fewer SRLGs a lightpath is associated with, the higher its reliability is likely to become.

Theorem 3.1: Without loss of generality, let the non-failure probabilities of all SRLGs in G be sorted in descending order, i.e., $p_1 > p_2 > \ldots > p_{M,}$, and $M = |S|$.

For two lightpaths P_1 with reliability r_1 and P_2 with reliability r_2, let S_1^P be the set of all SRLGs to which the links of P_1 belong and let S_2^P be the set of all SRLGs to which the links of P_2 belong.

Then a sufficient condition for $r_1 > r_2$ is: (C3.1)

$|S_1^P| < |S_2^P|$ and $\prod_{i=1}^{k} p_i < \prod_{i=M-k+2}^{M} p_i$ for all $1 < k < (M+2)/2$

Proof:

Let p_i be the non-failure probability of SRLG s_i^P

From Eq. 1,

$$r_1 = \prod_{s_i^P \in S_1^P} p_i \geq \prod_{i=M-|S_1^P|+1}^{M} p_i$$

$$r_2 = \prod_{s_i^P \in S_2^P} p_i \leq \prod_{i=1}^{|S_2^P|} p_i$$

If $|S_2^P| < (M+2)/2$,

$$r_2 \leq \prod_{i=1}^{|S_2^P|} p_i < \prod_{i=M-|S_2^P|+2}^{M} p_i \,,$$

If $|S_2^P| \geq (M+2)/2$ and M is odd,

$$\prod_{i=1}^{|S_2^P|} p_i = \prod_{i=1}^{\lfloor (M+2)/2 \rfloor} p_i \prod_{i=\lceil (M+2)/2 \rceil}^{|S_2^P|} p_i$$

$$< \prod_{i=M-\lfloor (M+2)/2 \rfloor+2}^{M} p_i \prod_{i=\lceil (M+2)/2 \rceil}^{|S_2^P|} p_i$$

$$< \prod_{i=\lceil (M+2)/2 \rceil}^{M} p_i \prod_{i=M-|S_2^P|+2}^{\lfloor (M+2)/2 \rfloor} p_i$$

and

$$\prod_{i=\lceil (M+2)/2 \rceil}^{M} p_i \prod_{i=M-|S_2^P|+2}^{\lfloor (M+2)/2 \rfloor} p_i = \prod_{i=M-|S_2^P|+2}^{M} p_i$$

Thus

$$r_2 < \prod_{i=M-|S_2^P|+2}^{M} p_i$$

If $|S_2^P| \geq (M+2)/2$ and M is even,

$$\prod_{i=1}^{|S_2^P|} p_i = \prod_{i=1}^{M/2} p_i \prod_{i=M/2+1}^{|S_2^P|} p_i$$

$$< \prod_{i=M-M/2+2}^{M} p_i \prod_{i=M/2+1}^{|S_2^P|} p_i$$

$$< \prod_{i=M/2+2}^{M} p_i \prod_{i=M-|S_2^P|+2}^{M/2+1} p_i = \prod_{i=M-|S_2^P|+2}^{M} p_i$$

and

$$\prod_{i=M/2+2}^{M} p_i \prod_{i=M-|S_2^P|+2}^{M/2+1} p_i = \prod_{i=M-|S_2^P|+2}^{M} p_i$$

Thus

$$r_2 < \prod_{i=M-|S_2^P|+2}^{M} p_i$$

Since $|S_1^P| < |S_2^P|$,

$$M - |S_1^P| + 1 \leq M - |S_2^P| + 2$$

Thus $r_1 > r_2$. □

For networks that satisfy C3.1, we propose heuristic solution (HA-3). In this solution, we run solution (A-1) in W of G's subnetworks, starting from the ones containing only the links that belong to a single SRLG, then going to the subnetworks containing the links that belong to two SRLGs, then three SRLGs, and so on. If a lightpath between s and t is found in any of the subnetworks, the lightpath is returned and the algorithm stops. The value of W is user-configurable. The details are as follows.

Algorithm (HA-3)

Step 1: *sort the SRLGs in descending order of their non-failure probabilities; let the sorted SRLGs be S_1, S_2, ..., S_M;*

Step 2: *ct = 0;*
for (k = 1, 2, ..., M) {
    for (each subnetwork that contains only the links belonging to SRLGs S_{i_1}, S_{i_2}, ..., S_{i_k} and $i_1 = 1, 2, ..., M-k+1$, $i_2 = i_1, i_1+1, ..., M-k+2$, ..., $i_k = i_1+k-1, i_1+k, ..., M$) {
        run (A-1);
        if (succeeds) return the lightpath and quit;
        if (ct equals W)
            print "No path found" and quit;
        increment ct by 1;
    }
}

The user-configurable value of W determines the number of times for which algorithm (A-1) is executed in Step 2. Thus the running time is $O(W|N|log|N|)$.

Now let δ be the maximum difference of the non-failure probabilities of any two consecutive SRLGs in the list obtained in Step 1 of (HA-3).

Theorem 3.2: The lightpath generated by (HA-3) is an $O(Mp_1^{k-1}\delta)$-approximation of the optimal lightpath where p_1 is the non-failure probability of SRLG S_1.

Proof: For networks that satisfy C3.1, from Theorem 3.1, if (HA-3) generates a lightpath in a subnetwork containing k

SRLGs, the path has the lowest reliability when the k SRLGs are $S_{M-k+1}, \ldots, S_{M-1}, S_M$, and the path has the highest reliability when the k SRLGs are $S_1, S_2, \ldots,$ and S_k.

Thus for the reliability r of the path,

$$\prod_{i=M-k+1}^{M} p_i \le r \le \prod_{i=1}^{k} p_i$$

Thus the maximum difference between r and the reliability of the optimal path r_0 is

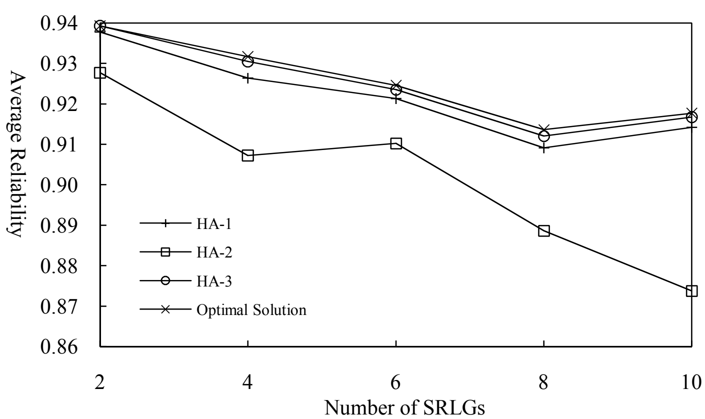
$$r_0 - r < \prod_{i=1}^{k} p_i - \prod_{i=M-k+1}^{M} p_i < p_1^k - (p_1 - M\delta)^k = O(Mp_1^{k-1}\delta)$$

□

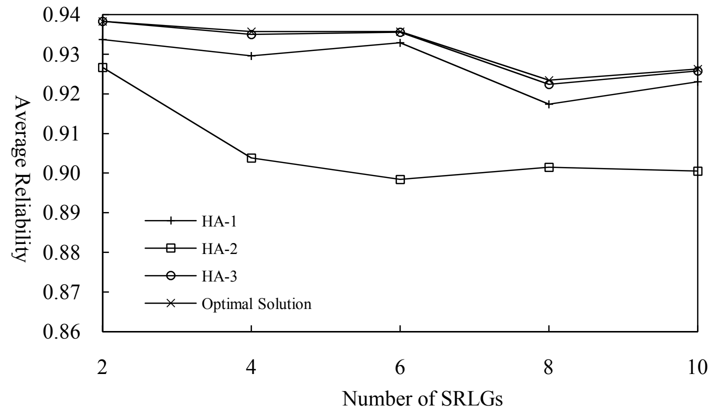
4. SIMULATIONS

We used LEDA programs to randomly generate network graphs for the simulations [22]. The network size ranged from 10 to 40 nodes. The nodal degree ranged from 2.6 to 3.0. The number of SRLGs in each network ranged from 2 to 10. The non-failure probabilities of the SRLGs were uniformly distributed in the range from 0.9100 to 0.9700. Each fiber link was assumed to support an unlimited number of lightpaths and was assumed to belong to any SRLG with equal probability.

The heuristic algorithms (HA-1), (HA-2) and (HA-3) were evaluated. For all pairs of end nodes, we ran the algorithms to obtain the maximum reliable lightpaths. We then compared the average reliabilities of the lightpaths. For (HA-3), we chose values for W such that it successfully generated paths. To generate the optimal solutions for comparison purpose, we exhaustively executed (A-1) for the subnetworks that contained all non-empty subsets of the SRLG set of the original networks. Two sets of the results are depicted in Fig. 2 and Fig. 3. Results from other network topologies are similar.

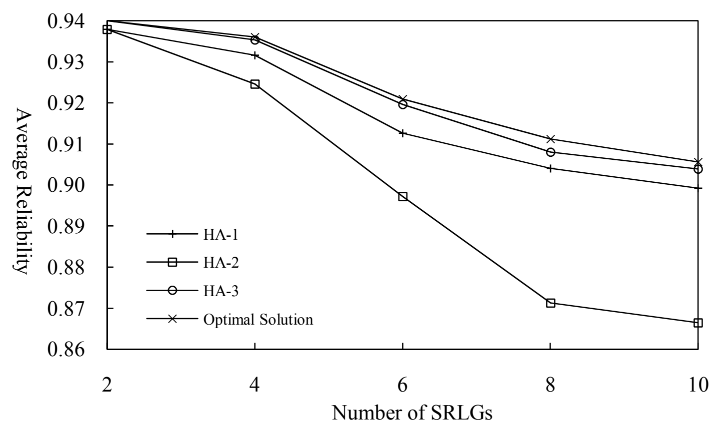


a. Nodal degree = 2.6

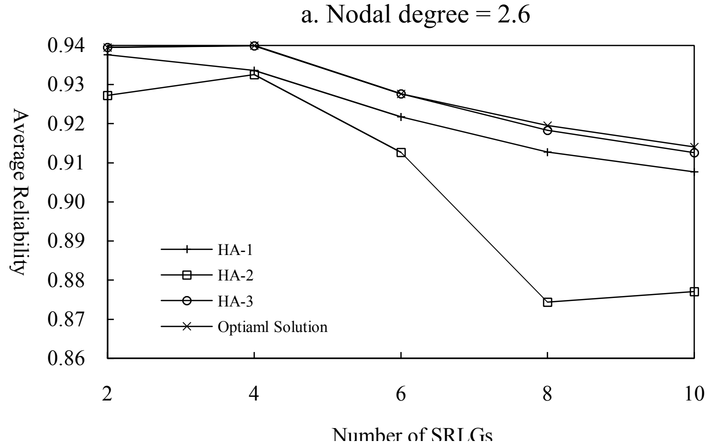


b. Nodal degree = 3.0

Fig. 2. Average lightpath reliability vs. Number of SRLGs. Number of nodes = 20



a. Nodal degree = 2.6



b. Nodal degree = 3.0

Fig. 3. Average lightpath reliability vs. Number of SRLGs. Number of nodes = 40.

We observed that the amount of time for the algorithms to generate the reliable lightpaths was small. For instance, to generate 780 lightpaths between every pair of nodes in a 40-node network with 3.0 nodal degree and 10 SRLGs, it took less than a second for solutions (HA-1) and (HA-2), and about 15 seconds for solution (HA-3) on a personal computer with a dual-core Intel processor. As a comparison, the optimal solution took about 30 seconds.

The simulations demonstrate that the performances of (HA-1) and (HA-3) are close to that of the optimal solution. Heuristic solution (HA-3) in particular, generates lightpaths with reliabilities that are indistinguishable from those of the optimal lightpaths. On the other hand, the performance of solution (HA-2) is less than satisfactory.

We note that the number of SRLGs and their non-failure probabilities have a significant impact on the average reliabilities of the lightpaths. The reliabilities improve along with the increase of the average non-failure probabilities of the SRLGs. In addition, when the number of SRLGs is small, a lightpath is more likely to consist of fiber links that belong to fewer SRLGs. Consequently, from (Eq. 1), the lightpath has higher reliability if the non-failure probabilities of the SRLGs do not vary significantly. As the number of SRLGs increases, the fiber links of a lightpath are more likely to belong to a bigger variety of SRLGs, which reduces the reliabilities of the lightpaths.

We also note that, as the nodal degree increases, the average reliabilities of the lightpaths increase. The reason for this behavior is that an increase in nodal degree results in higher average number of fiber links belonging to each SRLG, thereby making a lightpath more likely to consist of fiber links that belong to fewer SRLGs.

The network topology and the size of the network also affect the reliabilities of the lightpath. Larger networks with more nodes result in a higher average hop count for lightpaths; hence, for the same nodal degree and the number of SRLGs, the fiber links of the lightpaths in a network with more nodes will belongs to a greater number of SRLGs than lightpaths in a network with fewer nodes, which reduces the reliabilities of the lightpaths in a larger network.

5. Conclusions

In this paper we discussed the problem of finding the maximum reliable lightpath between two nodes under multiple failures. We proposed three heuristic solutions to solve the problem. Computer simulation demonstrated that solution (HA-3) generates results that are extremely close to the optimal. It also showed that the results from solution (HA-1) are less than optimal but its execution is very fast. On the other hand, solution (HA-2) which is based on Chvatal's $O(log|N|)$-approximation algorithm performs less than satisfactory.

From the simulations, we also observed that various factors affect the reliabilities of the lightpaths, including the nodal degree, the number of SRLGs, the non-failure probabilities of individual SRLGs, and the number of nodes in the network. An increase in the nodal degree helps increase the reliabilities of the lightpaths. This is due to the fact that there is a greater choice of routes for the lightpaths. The number of SRLGs also affects the reliabilities of the lightpaths. If the non-failure probabilities of the SRLGs are close to each other, an increase in the number of SRLGs reduces the reliabilities of the lightpaths. Larger number of network nodes also reduces the reliabilities of the lightpaths since the lightpaths are now longer and hence are more likely to contain more SRLGs.

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